INFERENCE OF SEISMIC GROUND MOTION
BY AUTOCOVARIANCE FUNCTIONS

by

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SYNOPSIS

A method is presented for estimating the root mean square distribution of acceleration, velocity and strain with subsurface depth during earthquakes, from seismic records obtained at the ground surface. The method is based on the multiple reflection theory of waves in layered ground and utilizes the relationship between the root mean square value at an arbitrary depth and the autocovariance function of the ground surface motion.

The procedure is applied to the El Centro (1940), Taft (1950) and Tokachi-oki (1968) earthquake records and a rough estimate of the maximum acceleration and strain in the subsurface during these earthquakes is made.

INTRODUCTION

A great number of strong motion seismographs have been installed and are in operation in the active earthquake zones of the world. These instruments are capable of supplying accelerograms of local, destructive strong motion earthquakes. Even though strong motion accelerograms are being accumulated steadily in this way, the total number obtained to date is still relatively small.

Some of the more important records have been used as input for structural response studies. However, it is recognized that most accelerograms recorded at the ground surface are modified by near-surface soil layers and that an interaction between structures and the subsurface may be induced, especially in the case of important and heavy structures which usually have deep foundations. A number of studies have been performed relating to the estimation of wave forms of motion in the subsurface and they have contributed to an understanding of the problem of the response of multi-layered ground to earthquakes, and to so-called inverse problems (1, 2, 3).

Considering the scarcity of destructive earthquake records, it is of the greatest importance to extract the maximum amount of information from those accelerograms in which the seismic characteristics and physical properties of the ground are necessarily reflected. While it is well known that during earthquakes the physical properties of the soil and surrounding structures depend largely on the stress and strain levels induced in the ground, scarcely any information exists regarding the strain levels actually reached. From this point of view, the measurement and/or inference of stress or strain developed in the subsurface is an area of research deserving of more attention.

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Because of the difficulty of measuring actual strains in the subsurface, it is particularly important to develop analytical procedures for inferring the strains produced during actual earthquakes. If this information can be obtained from a number of important existing accelerograms, as well as from those which will become available in the future, it should provide useful date for theoretical and experimental studies related to soil properties, and for structural and ground response investigations.

In the present paper the relationship between the root mean square value of underground motion and the autocovariance function of the motion at ground surface is deduced and it is expanded to an inference of the r.m.s. distribution of acceleration, velocity and strain with depth. These formulations are then applied to nine strong motion accelerograms obtained at six stations. Some important conclusions are deduced from these numerical computations.

ROOT MEAN SQUARE REPRESENTATION OF GROUND MOTION

Let f(t) be an ascending wave at depth z in a semi-infinite half space as shown in Fig.1. Under the assumption of one-dimensional elastic wave propogation, the motion at the ground súrface and at the depth z, $w_s(t)$ and $w_z(t)$ respectively, is written as follows:

$$w_s(t) = 2\,f(t - z/c) \qquad (1)$$

$$w_z(t) = f(t) + f(t - 2\,z/c) \qquad (2)$$

where c represents the velocity of the travelling waves.

Emphasis will hereafter be placed on the root mean square and/or variance of wave motion at various depths in the subsurface. The variance σ_s^2 of the record $w_s(t)$, obtained at the ground surface over the time period T is defined by the following equation:

$$\sigma_s^2 \equiv \frac{1}{T}\int_o^T w_s^2(t)\,dt \qquad (3)$$

The variance of $w_z(t)$ for the corresponding period is similarly expressed:

$$\sigma_z^2 = \frac{1}{T}\int_o^T \{f(t) + f(t - 2\,z/c)\}^2\,dt \qquad (4)$$

From Eq.(1) we have

$$f(t) = \frac{1}{2}\,w_s(t + z/c) \qquad (5)$$

Substitution of this equation into Eq.(4) yields

$$\sigma_z^2 = \frac{1}{4T}\left\{\int_o^{T-z/c} w_s^2(t + z/c)dt + \int_{z/c}^{T} w_s^2(t - z/c)dt\right.$$

$$\left. + 2\int_{z/c}^{T-z/c} w_s(t + z/c)\, w_s(t - z/c)dt\right\}$$

Shifting the time base of each term leads to

$$\sigma_z^2 = \frac{1}{4T}\left\{\int_{z/c}^{T} w_s^2(t)dt + \int_o^{T-z/c} w_s^2(t)dt\right.$$

$$\left. + 2\int_{2z/c}^{T} w_s(t)\, w_s(t - 2z/c)dt\right\} \qquad (6)$$

In these expressions z/c represents the wave travel time between the ground surface and depth z. From the engineering point of view, the depth to be considered is of the order of about one hundred meters and, therefore, the ratio z/c is very small compared to the time duration for usual earthquake records. That is

$$z/c \ll T \qquad (7)$$

The first term on the right hand side of Eq. (6) corresponds to the variance of a record from which the first z/c time interval is missing, and the second term corresponds to a record from which the last z/c time interval is missing. The low amplitude portion at the beginning and end of an earthquake record with long time duration contributes very little to the variance of that record. From these considerations the following approximation becomes possible:

$$\frac{1}{T}\int_{z/c}^{T} w_s^2(t)dt \doteqdot \frac{1}{T}\int_o^{T-z/c} w_s^2(t)dt \doteqdot \sigma_s^2 \qquad (8)$$

While the last term of the right hand side of Eq. (6) represents the double of the cross-correlation function between the ascending wave and the descending wave at depth z, it is regarded as the autocovariance function of the incident wave f(t) itself, because the ascending wave does not change its wave form by reflection at the ground surface. Therefore,

$$\frac{1}{T}\int_{2z/c}^{T} w_s(t)\, w_s(t - 2z/c)dt \doteqdot \frac{1}{T-2z/c}\int_{z/c}^{T-z/c} w_s(t)\, w_s(t - 2z/c)dt$$

$$\equiv \phi_s(2z/c) \qquad (9)$$

where $\phi_s(2z/c)$ represents the autocovariance function of record $w_s(t)$.

From Eqs. (8) and (9), Eq. (6) can be rewritten as follows:

$$\sigma_z = \frac{1}{\sqrt{2}} \sqrt{\sigma_s^2 + \phi_s\,(2z/c)} \tag{10}$$

This equation implies that the root mean square value of the seismic motion at depth z is completely inferred from the autocovariance function ϕ_s (τ) of the earthquake record which is observed at the ground surface, provided the condition of Eq. (7) is valid. Since σ_z is a function of the depth z, which is arbitrary, we can infer the vertical distribution of the r.m.s. from the record obtained at the ground surface by making use of Eq. (10), provided the condition of Eq. (7) is valid. This important principle is basic to the remainder of the paper.

ACCELERATION DISTRIBUTION UNDERGROUND

The development of the preceding section was formulated on the basis of a semi-infinite half space. Even if the ground is multiple layered, these results are applicable to the first layer without any modification, because we can regard all waves travelling upwards as ascending waves regardless of the multiple reflections and refractions at the interface. Therefore, the vertical distribution of the r.m.s. of acceleration can be obtained by making use of Eq. (10) for depths not greater than the interface. However, if it is required to extend this method to depths beneath the interface, further formulation is still necessary and may be performed as follows.

Consider a homogeneous layer with uniform thickness H_1, which extends over a second layer of thickness H_2 as shown in Fig.2. Let f(t) be the incident ascending seismic wave at the bottom of the second layer. In this case the refraction of the wave is induced at the first interface and multiple reflections of the refracted wave are induced in the first layer.

Considering this reflection and refraction at the ground surface and the first interface, the motions in the first layer and the second layer, w_1 (z,t) and w_2 (z,t) respectively, are written as follows:

$$\begin{aligned} w_1(z,t) = {} & \gamma_2\, f(t - \frac{H_2}{c_2} - \frac{H_1 - z}{c_1}) + \gamma_2\, f(t - \frac{H_2}{c_2} - \frac{H_1 + z}{c_1}) \\ & + \gamma_2\beta_1\, f(t - \frac{H_2}{c_2} - \frac{2H_1}{c_1} - \frac{H_1 - z}{c_1}) \\ & + \gamma_2\beta_1\, f(t - \frac{H_2}{c_2} - \frac{2H_1}{c_1} - \frac{H_1 + z}{c_1}) \\ & + \gamma_2\beta_1^2\, f(t - \frac{H_2}{c_2} - \frac{4H_1}{c_1} - \frac{H_1 - z}{c_1}) \\ & + \gamma_2\beta_1^2\, f(t - \frac{H_2}{c_2} - \frac{4H_1}{c_1} - \frac{H_1 + z}{c_1}) + \ldots, \text{for } H_1 \geqq z \geqq 0 \end{aligned} \tag{11}$$

$$w_2(z,t) = f(t - \frac{H_2 + H_1 - z}{c_2}) + \beta_2 f(t - \frac{H_2 - H_1 + z}{c_2})$$

$$+ \gamma_1\gamma_2 f(t - \frac{H_2}{c_2} - \frac{2H_1}{c_1} + \frac{H_1 - z}{c_2})$$

$$+ \gamma_1\gamma_2\beta_1 f(t - \frac{H_2}{c_2} - \frac{4H_1}{c_1} + \frac{H_1 - z}{c_2})$$

$$+ \gamma_1\gamma_2\beta_1^2 f(t - .....) + ..., \text{for } H_2 \geq z \geq H_1 \qquad (12)$$

In these equations, β_1, β_2, γ_1 and γ_2 are, respectively, reflection coefficients and refraction coefficients which are given by

$$\beta_1 = \frac{1 - \alpha}{1 + \alpha}, \quad \beta_2 = \frac{\alpha - 1}{1 + \alpha}, \quad \gamma_1 = \frac{2}{1 + \alpha}, \quad \gamma_2 = \frac{2\alpha}{\alpha + 1} \qquad (13)$$

where $$\alpha = \frac{\rho_2 c_2}{\rho_1 c_1} \qquad (14)$$

and ρ and c are densities and wave velocities which specify the layers.

Introducing an expression F(t) which is given by the following equation:

$$F(t) = \gamma_2 \{ f(t - \frac{H_2}{c_2} - \frac{H_1}{c_1}) + \beta_1 f(t - \frac{H_2}{c_2} - \frac{3H_1}{c_1})$$

$$+ \beta_2^2 f(t - \frac{H_2}{c_2} - \frac{5H_1}{c_1}) + ... \},$$

$$= \gamma_2 \sum_{n=o}^{\infty} \beta_1^n f\{t - \frac{H_2}{c_2} - \frac{(2n + 1)H_1}{c_1}\} \qquad (15)$$

Eqs. (11) and (12) reduce to:

$$w_1(z,t) = F(t + \frac{z}{c_1}) + F(t - \frac{z}{c_1}), \qquad (16)$$

$$\text{for } H_1 \geq z \geq 0$$

$$w_2(z,t) = f(t - \frac{H_2 + H_1 - z}{c_2}) + \beta_2 f(t - \frac{H_2 - H_1 + z}{c_2}$$

$$+ \gamma_1 F(t - \frac{H_1}{c_1} + \frac{H_2}{c_2} - \frac{z}{c_2}), \text{for } H_2 \geq z \geq H_1 \qquad (17)$$

Eq. (16) is obviously equivalent to the set of Eqs. (1) and (2). Thus, this is the basis for applying the results of the previous section to the first layer, independently of the motion of the second layer. Following the same procedure as that of the previous section, the r.m.s. distribution $\sigma_1(z)$ for the first layer is obtained, namely

$$\sigma_1(z) = \frac{1}{\sqrt{2}} \sqrt{\sigma_s^2 + \phi_s\,(2z/c_1)} \qquad \text{for } H_1 \geqq z \geqq 0 \tag{18}$$

where σ_s^2 and ϕ_s (τ) represent, respectively, the r.m.s. and autocovariance function of the record obtained at the ground surface. By use of the above equation the vertical distribution of acceleration in the first layer is easily obtained from the autocovariance function alone, which also gives σ_s^2.

Next, a similar expression for the r.m.s. distribution in the second layer is derived from Eqs. (15) and (17). By virtue of $|\beta_1| < 1$ the following is deduced from Eq. (15)

$$F(t + \frac{H_2}{c_2} + \frac{H_1}{c_1}) - \beta_1 F(t + \frac{H_2}{c_2} - \frac{H_1}{c_1}) = \gamma_2\, f(t) \tag{19}$$

Substitution of this equation into Eq. (14) yields

$$\begin{aligned} w_2(z,t) = \frac{1}{\gamma_2}\{&F(t + \frac{H_1}{c_1} - \frac{H_1}{c_2} + \frac{z}{c_2}) + F(t - \frac{H_1}{c_1} + \frac{H_1}{c_2} - \frac{z}{c_2}) \\ &- \beta_1 F(t - \frac{H_1}{c_1} - \frac{H_1}{c_2} + \frac{z}{c_2}) + \beta_2 F(t + \frac{H_1}{c_1} + \frac{H_1}{c_2} - \frac{z}{c_2})\} \end{aligned}$$

$$\text{for } H_2 \geqq z \geqq H_1 \tag{20}$$

in which the relationship $\gamma_1\gamma_2 - \beta_1\beta_2 = 1$ is used.

Writing $z' = z - H_1$, Eq. (20) yields

$$\begin{aligned} \gamma_2 w_2(z',t) = &F(t + \frac{H_1}{c_1} + \frac{z'}{c_2}) + \beta_2 F(t + \frac{H_1}{c_1} - \frac{z'}{c_2}) \\ &- \beta_1 F(t - \frac{H_1}{c_1} + \frac{z'}{c_2}) + F(t - \frac{H_1}{c_1} - \frac{z'}{c_2}) \end{aligned}$$

$$\text{for } H_2 \geqq z' \geqq 0 \tag{21}$$

Taking the mean square of $w_2(z,t)$ over the time period T of the record, and assuming that H_1/c_1 and z'/c_2 are negligible compared with T, leads to

$$\gamma_2^2\,\sigma_2^2(z') = (2 + \beta_1^2 + \beta_2^2)\sigma_F^2$$

$$+ 2\{(\beta_2 - \beta_1)[\phi_F(2z'/c_2) + \phi_F(2H_1/c_1)]$$

$$+ \phi_F(2H_1/c_1 + 2z'/c_2) - \beta_1\beta_2\phi_F(2H_1/c_1 - 2z'/c_2)\}$$

$$\text{for } H_2 \geqq z' \geqq 0 \qquad (22)$$

where σ_F^2 and ϕ_F represent respectively the variance and autocovariance of F(t). At the same time, the following relationship exists between the variance σ_F^2 of F(t) and that of $w_s(t)$, which is the record at the ground surface:

$$\sigma_F^2 = \frac{1}{4}\sigma_s^2$$

and for autocovariance

$$\phi_F(\tau) = \frac{1}{4}\phi_s(\tau)$$

Accordingly

$$\sigma_2(z') = \frac{1}{2\gamma_2}\sqrt{(2 + \beta_1^2 + \beta_2^2)\sigma_s^2 + 2\{(\beta_2 - \beta_1)[\phi_s(2z'/c_2) + \phi_s(2H_1/c_1)] + \phi_s(2H_1/c_1 + 2z'/c_2) - \beta_1\beta_2\phi_s(2H_1/c_1 - 2z'/c_2)\}}\ ,$$

$$\text{for } H_2 \geqq z' \geqq 0 \qquad (23)$$

By use of Eqs. (18) and (23), the acceleration distribution with subsurface depth, including the first and the second layers, can be computed from the autocovariance function of accelerograms obtained at the ground surface, provided the values of c_1, c_2 and H_1 are known. This method can be easily extended to multi-layered ground, but the reliability of the results decreases for deeper layers since the period of wave transmission between the ground surface and the depth concerned is no longer negligible compared with the record duration.

STRAIN DISTRIBUTION UNDERGROUND

Strain is defined, for practical purposes, as the ratio of the relative displacement of two points to the distance between them. It is possible to calculate strain during an earthquake if the displacement variations at various depths from the ground surface are known. However, the ground surface displacement curve, for instance, which is obtained by double integration of the accelerogram, is affected strongly by details of the data processing procedure, such as digitizing interval of the record, base line correction, duration time, integration method and so on. Moveover, the subtracting which is inevitable in the computation of the relative displacement between two points for a given time interval, necessarily causes a drop in the number of significant digits in the calculations. Therefore, the determination of strain by this method is not satisfactory for the present analysis.

While it is possible to observe the actual displacement curve at various depths by seismometers which are arranged a short distance from each other, it is nevertheless difficult and expensive to secure the required information experimentally. On the other hand, in problems of wave transmission, strain can be expressed directly in velocity terms. The necessary data for the purpose at hand can be readily obtained from accelerograms, which are less affected by the data processing procedure than are the displacement curves. In the present analysis it is not the strain curve itself that is dealt with but rather the r.m.s. of the strain; consequently, the subtraction procedure is not required. Using this approach, the strain distribution can be calculated from the ground surface velocity curve alone, and neither the double integration of accelerograms nor the subtraction procedure is required. Accordingly, the same method followed in the preceding section will also be applied to the problem of calculating the strain.

After replacing the acceleration w(z,t) of the previous discussion with the displacement u(z,t), and F(t) with U(t), the displacement can be written as follows:

$$u_1(z,t) = U(t + \frac{z}{c_1}) + U(t - \frac{z}{c_1}), \text{ for } H_1 \geqq z \geqq 0 \tag{24}$$

$$\gamma_2 u_2(z,t) = U(t + \frac{H_1}{c_1} + \frac{z'}{c_2}) + \beta_2 U(t + \frac{H_1}{c_1} - \frac{z'}{c_2}) - \beta_1 U(t - \frac{H_1}{c_1} + \frac{z'}{c_2}) + U(t - \frac{H_1}{c_1} - \frac{z'}{c_2}),$$

$$\text{for } H_2 \geqq z' \geqq 0 \tag{25}$$

in which U(t) is defined by the same expression as Eq. (15) with f(t) replaced by the incident displacement wave. Strain in the first layer is given by

$$\varepsilon_1(z,t) = \frac{\partial u_1}{\partial z} = \frac{1}{c_1}\{U'(t + \frac{z}{c_1}) - U'(t - \frac{z}{c_1})\},$$

$$\text{for } H_1 \geqq z \geqq 0 \tag{26}$$

$\varepsilon_2^*(z') = \overline{2c_2\gamma_2}\ \ v(2 + \beta_1^2 + \beta_2^2)\sigma_v^2 + 2\{(\beta_1 - \beta_2)[\phi_v(2z'/c_2)$

$\overline{- \phi_v(2H_1/c_1)] - \phi_v(2H_1/c_1 + 2z'/c_2) + \beta_1\beta_2\phi_v(2z'/c_2}$

where U'(x) represents the derivative of U(x). From Eq. (24), the displacement at the ground surface is

$$U_o(t) = 2U(t) \tag{27}$$

and the corresponding velocity is

$$v_o(t) = 2U'(t) \tag{28}$$

Substitution of Eq. (28) into Eq. (26) leads to

$$\varepsilon(z,t) = \frac{1}{2c_1} \{v_o(t + \frac{z}{c_1}) - v_o(t - \frac{z}{c_1})\} , \quad \text{for } H_1 \geqq z \geqq 0 \tag{29}$$

Following the same procedure mentioned previously, the r.m.s. expression $\varepsilon_1^*(z)$ of strain in the first layer is given as follows:

$$\varepsilon_1^*(z) = \frac{1}{\sqrt{2}c_1} \sqrt{\sigma_v^2 - \phi_v(2z/c_1)} , \quad \text{for } H_1 \geqq z \geqq 0 \tag{30}$$

where σ_v^2 and $\phi_v(\tau)$ are, respectively, the variance and autocovariance function of the velocity record at the ground surface.

The autocovariance function $\phi_v(\tau)$ takes a value between σ_v^2 and $-\sigma_v^2$, i.e.,

$$\sigma_v^2 \geqq \phi_v(\tau) \geqq -\sigma_v^2 \tag{31}$$

Accordingly, the upper bound of the r.m.s. of strain in the first layer is given by

$$\varepsilon_u = \frac{\sigma_v}{c_1} \tag{32}$$

From this equation, a rough estimation of strain in the first layer is possible if the wave velocity in the layer and the velocity curve at the ground surface are given. Furthermore, it should be noted that this upper bound corresponds to the case of a sinusoidal wave, which gives a somewhat larger value of strain than the strain during an actual earthquake with the same σ_v.

The strain in the second layer will now be derived. Differentiation of Eq. (25) with respect to z' leads to

$$\gamma_2\varepsilon_2(z',t) = \frac{1}{c_2}\{U'(t + \frac{H_1}{c_1} + \frac{z'}{c_2}) - \beta^2 U'(t + \frac{H_1}{c_1} - \frac{z'}{c_2})$$

$$- \beta_1 U'(t - \frac{H_1}{c_1} + \frac{z'}{c_2}) - U'(t - \frac{H_1}{c_1} - \frac{z'}{c_2})\} ,$$

$$\text{for } H_2 \geqq z' \geqq 0 \qquad (33)$$

Substitution of Eq. (28) into Eq. (33) yields

$$2c_2\gamma_2\varepsilon_2(z',t) = v_o(t + \frac{H_1}{c_1} + \frac{z'}{c_2}) - \beta_2 v_o(t + \frac{H_1}{c_1} - \frac{z'}{c_2})$$

$$- \beta_1 v_o(t - \frac{H_1}{c_1} + \frac{z'}{c_2}) - v_o(t - \frac{H_1}{c_1} - \frac{z'}{c_2})$$

$$\text{for } H_2 \geqq z' \geqq 0 \qquad (34)$$

Applying the same procedure used above; the r.m.s. expression for strain in the second layer, $\varepsilon_2^*(z')$, is given as follows:

$$\varepsilon_2^*(z') = \frac{1}{2c_2\gamma_2} \sqrt{(2 + \beta_1^2 + \beta_2^2)\sigma_v^2 + 2\{(\beta_1 - \beta_2)[\phi_v(2z'/c_2) - \phi_v(2H_1/c_1)] - \phi_v(2H_1/c_1 + 2z'/c_2) + \beta_1\beta_2\phi_v(2z'/c_2 - 2H_1/c_1)\}} ,$$

$$\text{for } H_2 \geqq z' \geqq 0 \qquad (35)$$

Eq. (30) and Eq. (35) can be used to calculate the vertical distribution of strain, which is expressed in the r.m.s., when the velocity curve at the ground surface and values of c_1, c_2, p_1, p_2 and H_1 are known.

MAGNIFICATION FACTOR IN THE SURFACE LAYER

It is sometimes convenience to introduce the concept of the magnification factor as an expression of the seismic characteristics of the ground. The magnification factor is defined as the ratio of the r.m.s. of the acceleration curve at the ground surface to that at the interface between the first and the second layers. Hence, as defined above, the magnification factor represents the amplification in the first layer when acceleration at the two named points are evaluated in the r.m.s.

This definition of the magnification factor may be expressed mathematically as

$$\text{M.F.} = \frac{1}{\sigma_1(H)} \tag{36}$$

We may also write the following well-known relationships

$$\phi_s(0) = \sigma_s^2, \ \phi_s(\tau) = \sigma_s^2 C_s(\tau) \tag{37}$$

and $\quad \sigma_1(0) = \sigma_s$

where $C_s(\tau)$ represents the autocorrelation coefficient function.

Considering Eqs. (18) and (37), the magnification factor, M.F., is finally given by the following expression

$$\text{M.F.} = \sqrt{\left\{\frac{2}{1 + C_s(2H_1/c_1)}\right\}} \tag{38}$$

When the thickness, H_1, and the wave velocity in the first layer are known, the magnification factor in the layer is readily obtained from Eq. (38) by finding the value of the autocorrelation coefficient corresponding to $2H_1/c_1$. This is independent of the magnitude of the record because the autocorrelation coefficient is a normalized function.

Data about H_1 and c_1 are not necessarily available at every strong-motion earthquake recording station. In cases where these data are missing, the magnification factor may be estimated by the autocorrelation coefficient function provided the ground surface record shows evidence of layer amplification and the predominant period of the layer is clearly confirmed. In such cases, the value of the autocorrelation coefficient function for lag time $2H_1/c_1$ will be given by the corresponding value of the first negative peak in the coefficient function; this is so because $2H_1/c_1$ is one half of the predominant period to which the first positive peak corresponds. The utility of this method will be shown later for some particular earthquake records.

APPLICATION TO STRONG MOTION EARTHQUAKE RECORDS

Records The results obtained in the preceding sections were applied to some strong motion earthquakes recorded at five stations in the U.S.A. and Japan. These are El Centro (1940), Taft (1952) and three records obtained at different sites during the 1968 Tokachi-oki earthquake. The site characteristics of the observation stations are shown in Fig.3 and Fig.4. These data are taken from existing records of site characteristics available in the literature (4, 5, 6).

While the thickness of every layer and the wave velocity in each layer were measured or estimated and reported for the sites at El Centro and Taft, a uniform representation of these data for the three Japanese stations, Miyako, Muroran and Hachinohe, has not yet been reported in the literature. However, since the ground conditions describing these stations are available in the form shown in Fig.4, a rough estimation of the S wave velocity is possible by utilizing the relationship between the S wave velocity, v_s, and the N value of the standard penetration test.

Table 1 shows the referred and estimated site data used in the numerical computations. For Hachinohe, Miyako and Muroran, three sets of wave velocity and density were assumed in each layer for comparative purposes. Table 2 is a summary list of the basic earthquake data treated. The digitized acceleration records were taken from data provided in the literature (7, 8).

The three records for Hachinohe, Miyako and Muroran are reproduced in Fig.5 and the computed autocorrelation coefficient functions from these records are shown in Fig.6. Reproduction of records for El Centro and Taft were omitted because they are well known. While the original records for El Centro and Taft have been digitized at unequal time intervals, corresponding to all significant peaks and points of inflection, a linear interpolation between points was applied to convert the data into equal time intervals of 0.02 seconds. This occasionally causes a slight difference in maximum accelerations between the original and reproduced records. On the other hand, the records for Hachinohe, Miyako and Muroran have been digitized at equal time intervals of 0.01 seconds; the acceleration at every second time interval of the original record was taken as the acceleration for the 0.02 second intervals used in the calculation.

The parabolic-type base line correction (9) was applied to all accelerograms. Although a base line correction causes little difference to the acceleration itself, it can produce significant effects on the velocity and displacement curves when they are obtained through an integration of the accelerogram. Hence, while some base line correction is necessary when securing the velocity curves, from which the r.m.s. distribution of velocity and strain are derived, it is now always necessary to provide the correction when calculating the r.m.s. distribution of acceleration in the subsurface.

El Centro Fig.7 shows the r.m.s. distribution of acceleration, velocity and strain with depth, which were computed by the method mentioned previously. The S wave velocity of the site was used for the two horizontal components and the P wave velocity was used for the vertical component. The first graph of this figure illustrates that the magnification of acceleration amplitude is concentrated in the near-surface region, not deeper than about 10 meters. It is also evident that the difference between the two horizontal components decreases in proportion to the depth from the ground surface. Both of these components are magnified about 1.5 times in the short distance of 15 ~ 20 meters.

For the vertical component, the second mode seems to predominate in the first layer of the depth if the first interface and the wave velocity in the layer are appropriate. However, if the near-surface S wave velocity is reasonable for the site and the depth of the interface is not so, it could be assumed that another interface exists at about 10 meters depth. However, this supposition should be abandoned if the predominant frequency observed in the record at the surface is inherent in the incident wave. It may be seen that the magnitude of the vertical component is comparable with that of the horizontal components; however, it should be stated that the most intensive 12 seconds were used for the vertical component studies, which means that the magnitude of this component is slightly exaggerated compared with the others.

The second graph compares the r.m.s. distribution of velocity in the subsurface. Two notable differences are apparent when compared with the r.m.s. distribution of acceleration. The first is that the change of the r.m.s. amplitude with depth is not so remarkable as that observed for acceleration. The second difference relates to the small size of the vertical component when compared with horizontal components. The latter difference is due to the fact that the dominant frequency of the vertical component is higher than that of horizontal components, the explanation for the former difference is that the frequency band, which would be magnified in the first layer, is scarcely included in the velocity curve. Therefore, although the velocity curve is suitable to predict the motion of the deeper regions, the acceleration curve is adequate for regions near the surface. These tendencies are further exaggerated in the r.m.s. distribution of displacement in which case no change of amplitude is found with depth; a graphical illustration of this fact is omitted here.

The last graph of Fig.7 shows the strain distribution expressed in r.m.s. value; the r.m.s. of strain is of the order of 10^{-4}. The ratio of the maximum value to the r.m.s. cannot be presented in a deterministic expression because of the randomness of the earthquake motion. This ratio, which is discussed later, is usually about 3 to 5 for strong motion earthquake records with long duration periods. Therefore a rough estimation of the maximum shear strain is of the order of 10^{-3} or less. From this value of strain one can estimate the strain level in strong motion earthquakes of the intensity of El Centro 1940. Although the strain level within which the stress-strain relationship remains linear varies widely, depending on the type of soil, loading conditions, confining pressure, water content, type of travelling wave etc., it may be of the order of 10^{-4} to 10^{-3} (10, 11). Considering the fact that the El Centro earthquake is one of the most severe recorded earthquakes, and that the maximum value is an instantaneous but not a sustained value, it may be concluded that strain during earthquakes, even for strong motion earthquakes, does not necessarily exceed the linear range.

On the other hand, the vertical component is assumed to be the longitudinal wave in which the direction of particle movement is vertical. Therefore it produces not only compressive strain but also tensile strain. However the tensile strength of soil is, in general, considerably less than the compressive strength. Accordingly, if the tensile strain exceeds the linear range of the soil under consideration, this method, which is based on linear wave transmission, fails to be valid for the vertical component. This statement is also applicable for the shear strain due to the horizontal components.

<u>Taft</u> A general discussion on the vertical distribution of acceleration,

velocity and strain has already been given in terms of the results obtained from the El Centro records. In the acceleration distribution of Fig.8, the two curves for the horizontal components are very close to each other and both of the magnification factors between the interface and the ground surface are about 1.5.

Strain distributions for the horizontal components are also comparable despite the fact that these curves are calculated from the velocity records, for which the distribution curves take slightly different values, as shown in the graph. The reason for this is that the strain distribution is calculated not from the variance itself but from the difference between the variance and the autocovariance function. Consequently it is independent of the magnitude of the r.m.s. of velocity. The upper bound, ε_u, given in Eq. (32), coincides with the r.m.s. of strain at a depth which is equivalent to a quarter of the wave length dominant in the velocity curve. Values of ε_u for the two horizontal components in this example are, respectively, 356×10^{-6} for the N 69 W component and 300×10^{-6} for the other. Therefore, the estimation of strain in the subsurface during an earthquake by this upper bound may be said to be only very approximate.

Tokachi-oki Figs. 9, 10 and 11 illustrate the effect of wave velocity and density on the r.m.s. distribution of acceleration, velocity and strain. In the case of Hachinohe, no significant difference is found in the acceleration and velocity distribution for the three sets of wave velocity and density, however, the strain distribution is obviously inversely proportional to the wave velocity. If the thickness of the layer is definite, the Fourier spectrum is useful for estimating the wave velocity in the first layer. In the Fourier spectrum for Hachinohe there are four dominant peaks at approximately 0.4 c/s, 1.0 c/s, 1.6 ~ 2.8 c/s and 4.5 ~ 4.8 c/s. In view of the ground conditions, the first three peaks give wave velocities which are too large. The wave velocity of 192 m/sec., which corresponds to the last peak at about 4.8 c/s, may be considered as a reasonable value for the site.

For the Miyako site, the dominant frequency was readily found to be about 5.25 c/s. This leads to a 210 m/sec. wave velocity for the case of a 10 meters layer thickness. Therefore a wave velocity value of 200 m/sec. is considered to be a reasonable estimate for the first layer. The magnification factor in this case can be estimated directly by Eq. (38) without calculation of the r.m.s. distribution because it is probable that the predominant frequency is induced in the layer. A magnification factor of 2.5 is obtained when using the value corresponding to the first negative peak of the autocorrelation coefficient function shown in Fig.6.

Notwithstanding that the site characteristics of Muroran are close to those of Miyako, the amplification in the layer is now as pronounced as in the case of Miyako. This is mainly due to the frequency composition of the incident wave to the layer. That is, the dominant frequency of the record of Muroran is lower than the natural frequency of the layer. Usually, the possible range of estimated values of wave velocity is reasonably narrow provided the site characteristics are known, and any small changes in velocity produce only small changes in the distribution. Therefore, the inference of the seismic motion in the subsurface can probably be decided in almost all cases from the site characteristics of the recording station alone.

Maximum to Root Mean Square Ratio Although r.m.s. values rather than maximum values of a record have been treated throughout this paper, some relationship between these two quantities should be considered. If earthquakes or earthquake records are assumed to be a random process, there are some methods for obtaining this relationship by taking the ensemble average over the population. However, a single earthquake record cannot be considered to be a sample record from the population. In practice, therefore, the average on the time axis is used in place of the ensemble average and the value of this average depends on the averaging period. Table 3 shows the ratio of the maximum value to the r.m.s. value computed from each record used in this analysis; these values get larger for longer averaging periods.

In the method developed in this paper, the necessary time lag in the autocovariance function is equal to double the time taken for the wave to travel from the ground surface to the depth under consideration. This value, which for the two layer case is given by $2(H_1/c_1 + H_2/c_2)$, equals 0.360 seconds for El Centro and 0.284 seconds for Taft at 60 meter depth. Thus a time lag of 0.2 ~ 0.4 seconds may be sufficient for the analysis within 50 ~ 60 meter depth in usual near-surface materials. Furthermore, it could be said that the desirable record length for satisfying computations for the autocovariance function may be longer than twenty times the required lag time in the function. Therefore, the necessary record length in this analysis becomes about 5 ~ 8 seconds. Normal strong motion accelerograms are considered to be stationary over the most significant portion of the records for such time intervals. Accordingly, under the assumption that the seismic characteristics are the same in any portion of the record, it appears that a 5 ~ 8 second section of a record seems adequate as a replacement for an entire record when estimating the r.m.s. distribution to a depth of about 50 meters below the ground surface.

On the other hand, the moving average on the squared value of accelerograms has been successful for deducing the trend of intensity of an earthquake; the suitable average period is approximately equal to ten times the predominant period in the accelerograms (12). This period may fall within a range of 2 ~ 5 seconds for past strong motion accelerograms.

Fig.12 illustrates examples of the r.m.s. of a 5 second moving average period, which is normalized by the maximum value of the original record. In a sense these curves represent the fluctuation of the intensity of earthquakes. The reciprocals of the maximum values for all curves are listed in Table 4. These values are less than those given in Table 3, and also their range of scatter is seen to be considerably reduced; the values fall in the range 2.5 ~ 3.3.

These discussions are applicable also to strain. For confirmation, the strain curves at the interface for a few records were calculated from Eq. (29) and they are illustrated in Figs. 13 and 14. The maximum strain and the r.m.s. of the most intensive 5 seconds are 1.63×10^{-3} and 0.57×10^{-3} respectively for El Centro (1940), N-S component. The ratio of the maximum to the r.m.s. value in this case is 2.9. The ratios are also about 3 for the other records shown in Table 4 and they remain within the range mentioned above.

To summarize, approximate estimates of the ratio of the maximum value to the r.m.s. value of acceleration and strain distribution, for depths not greater than about 50 meters, will be in the range 2.5 - 3.3 provided the most significant and stationary portion of the record, which corresponds to a length of approximately ten times the predominant period, is chosen for the computations.

CONCLUSION

The present study demonstrates that the root mean square value of underground motion at an arbitrary depth can be expressed in terms of an autocovariance function of the earthquake record obtained at the ground surface. Thus the vertical distribution of underground motion and strain can be estimated from accelerograms at the ground surface and the physical constants of the ground alone.

The numerical computations indicate that for the example considered the magnification of acceleration in the subsurface is concentrated in the near-surface region, that is, not deeper than about 20 meters, while the velocity amplitude distribution shows no significant change with depth.

The maximum strain obtained from the El Centro earthquake record is roughly estimated to be in the order of 10^{-3}; the corresponding strains associated with the other earthquakes are of the same order of magnitude or less. These results suggest that the maximum strain level in strong motion earthquakes, excluding extremely destructive or near-epicentral earthquakes, does not necessarily induce non-linear behaviour of soil. However, this does not imply that a non-linear consideration is unnecessary in all situations. In particular, the strain analysis in the vicinity of structures is of importance in view of the interaction between the structure and ground; this problem remains unsolved.

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TABLE I

SITE CHARACTERISTICS

Site	First Layer				Second Layer		
	H_1	V_p	V_s	ρg	V_p	V_s	ρg
El Centro	19	360	157	2.0	1,770	843	2.08
Taft	12	357	160	2.24	1,500	720	2.30

Site	H_1	Case	First Layer		Second Layer	
			V_s	ρg	V_s	ρg
Hachinohe	10	1	190	1.6	380	2.0
		2	210	1.8	350	1.8
		3	160	1.7	400	1.9
Miyako	10	1	210	1.8	1400	2.7
		2	250	1.9	1200	2.6
		3	180	1.8	1300	2.6
Muroran	14	1	220	1.8	1100	2.5
		2	180	1.8	1200	2.6
		3	250	1.9	1000	2.5

H_1 : m, V_p and V_s : m/sec, ρg : gr/cm^3

TABLE II

DATA FOR EARTHQUAKE RECORDS TREATED

Station	Date	Component	Max. accel. (gal)	Record length (sec)
El Centro	May 18, 1940	NS	359	30.0
		EW	223	30.0
		UD	278	12.0
Taft	July 21, 1952	N 69 W	158	30.0
		S 21 W	176	30.0
		UD	123	30.0
Hachinohe	May 16, 1968	NS	235	30.0
Miyako	May 16, 1968	EW	95	30.0
Muroran	May 16, 1968	NS	209	35.0

TABLE III

MAX. TO R.M.S. RATIO FOR ENTIRE RECORD

Record		Accel.	Vel.
El Centro	NS	4.36	4.32
	EW	5.38	3.87
	UD	4.59	4.10
Taf·	N 69 W	4.46	4.01
	S 21 W	5.14	3.28
	UD	0.89	3.49
Hachinohe	NS	5.33	3.71
Miyako	EW	3.42	3.05
Muroran	NS	4.95	6.42

TABLE IV

MAX. TO R.M.S. RATIO FOR MOST INTENSIVE 5 SECONDS OF RECORD

Record		Accel.	Strain
El Centro	NS	2.93	2.88
	EW	2.77	3.14
	UD	3.62	-
Taft	N 69 W	2.60	3.09
	S 21 W	3.16	2.64
	UD	3.26	-
Hachinohe	NS	3.20	3.30
Miyako	EW	3.24	3.39
Muroran	NS	2.48	2.68

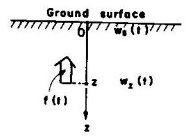

Fig. 1 Coordinate in Half Space

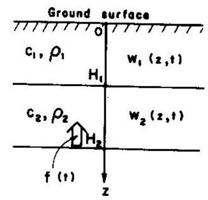

Fig. 2 Two Layer System

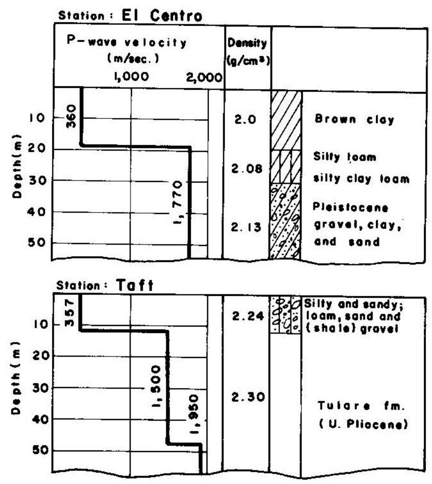

Fig. 3 Site Characteristics at Two Stations (El Centro, Taft)

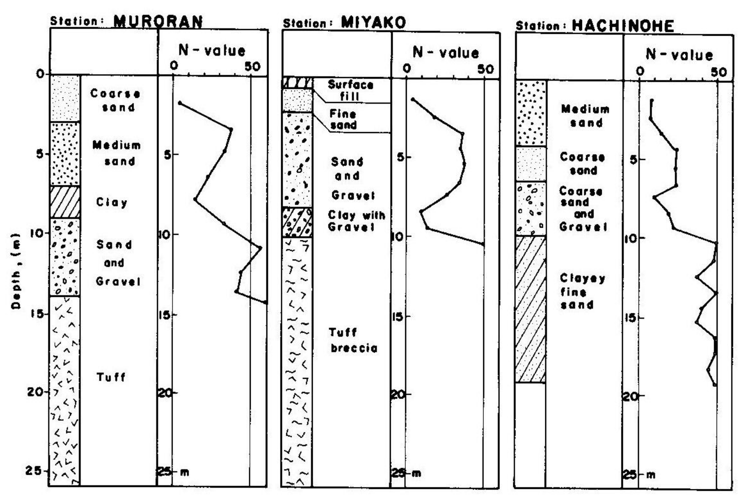

Fig. 4 Site Characteristics at Three Stations (Muroran, Miyako and Hachinohe)

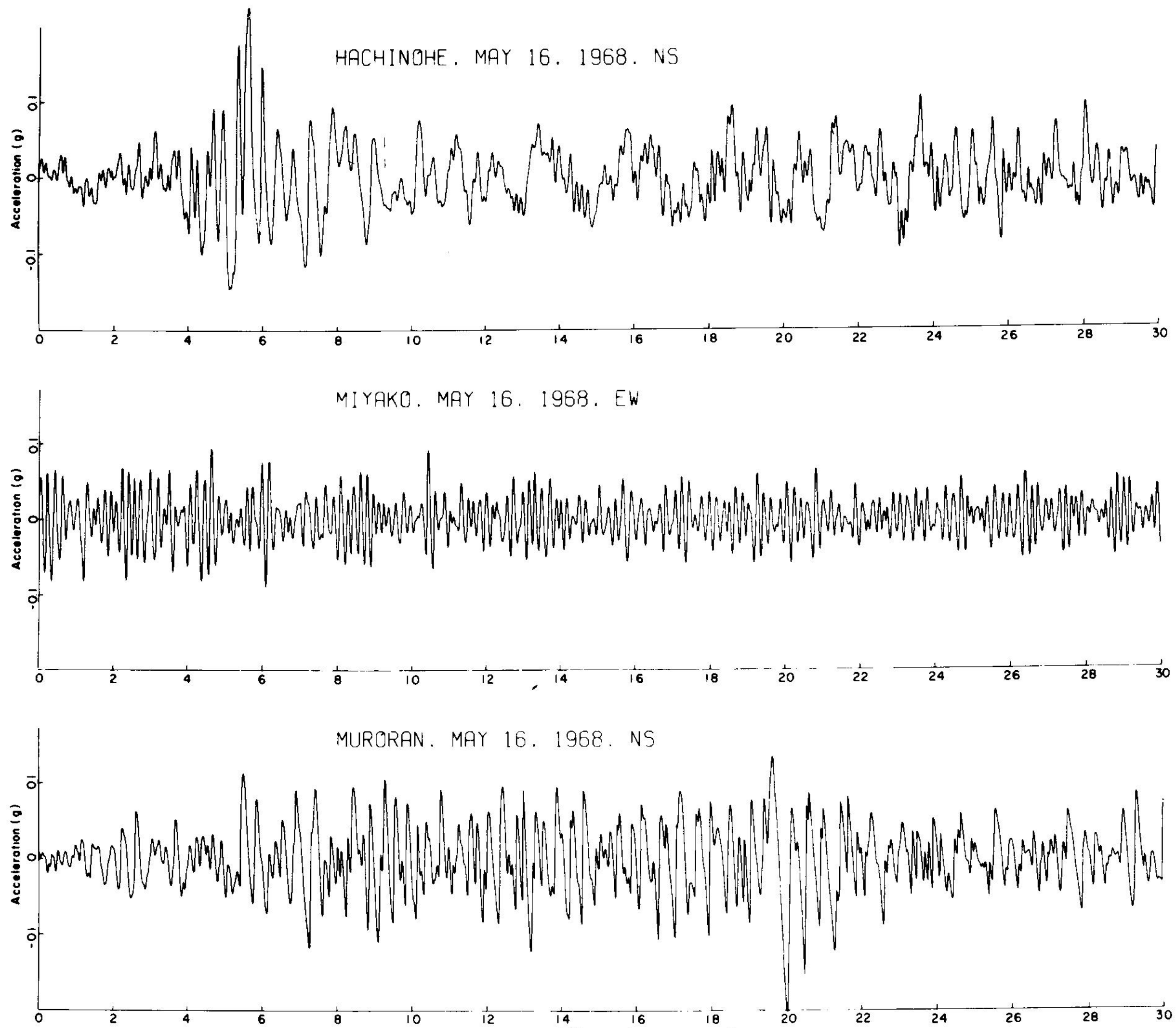

Fig 5 REPRODUCED ACCELEROGRAMS FOR HACHINOHE, MIYAKO AND MURORAN

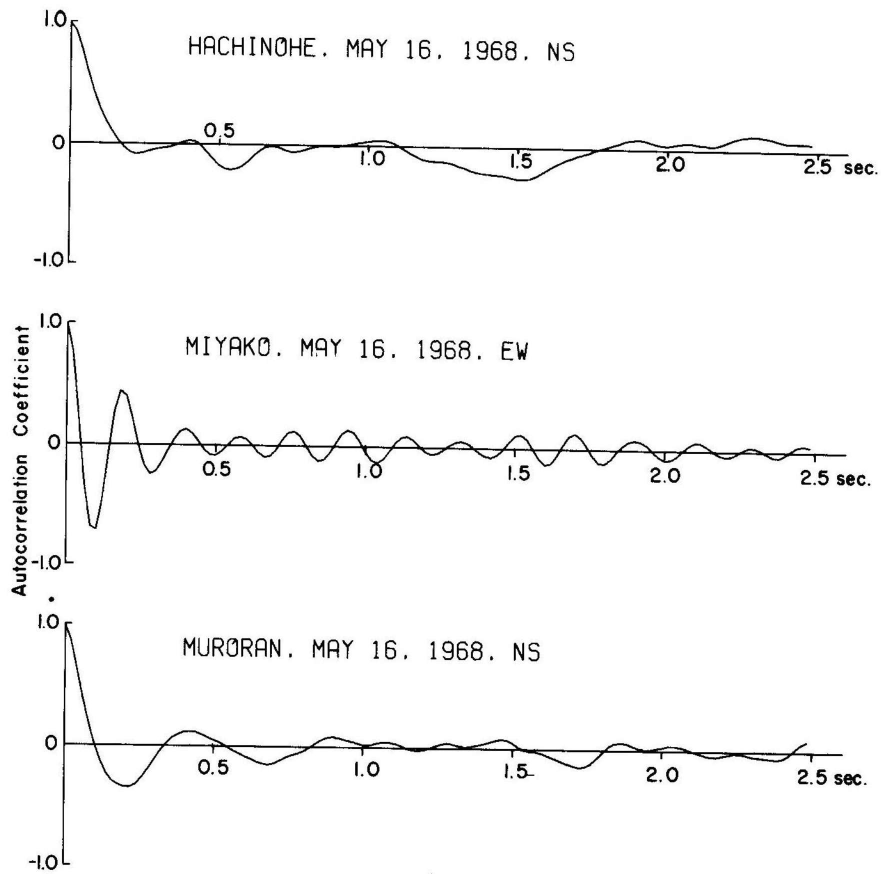

Fig. 6 Autocorrelation Coefficient Functions

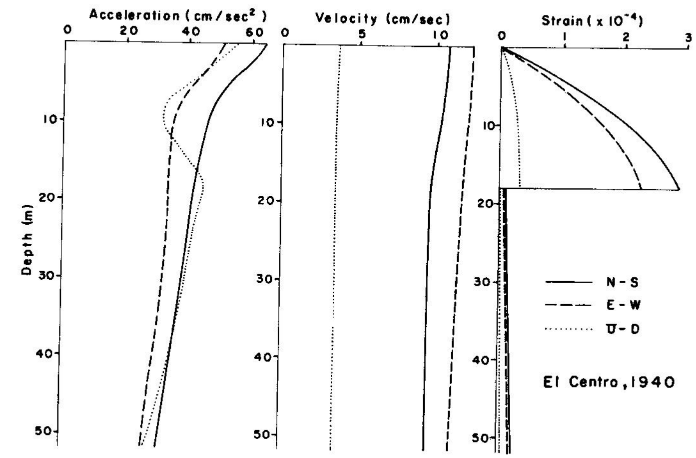

Fig. 7 R.M.S. Distributions with Depth (El Centro, 1940)

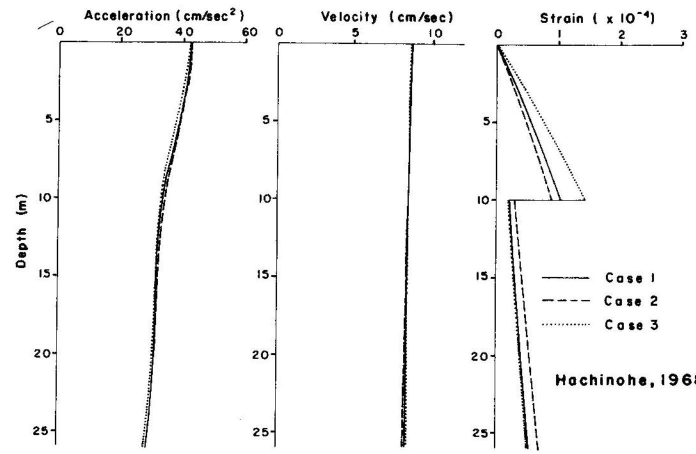

Fig. 9 R.M.S. Distributions with Depth (Hachinohe, 1968)

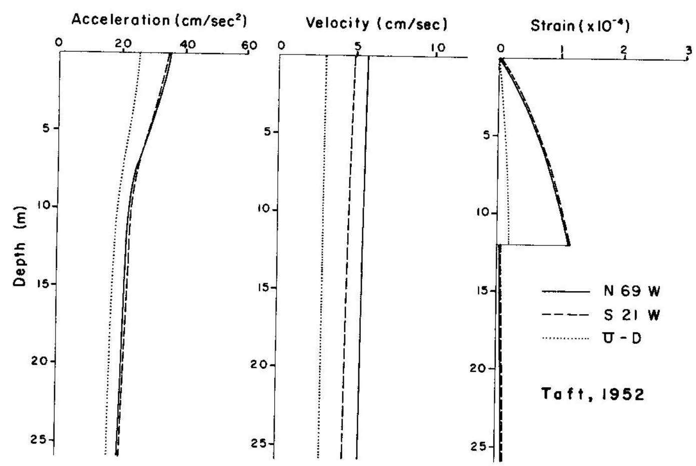

Fig. 8 R.M.S. Distributions with Depth (Taft, 1952)

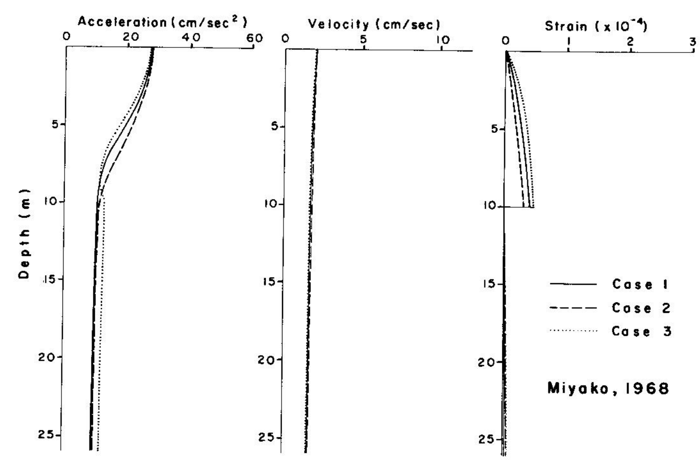

Fig. 10 R.M.S. Distributions with Depth (Miyako, 1968)

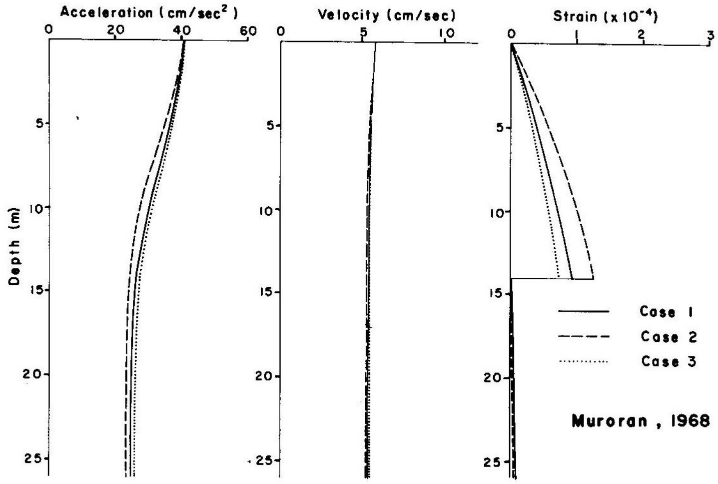

Fig. 11 R.M.S. Distributions with Depth (Muroran, 1968)

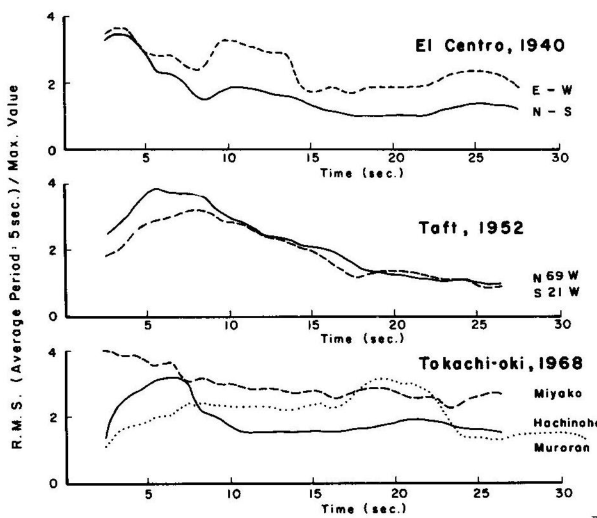

Fig. 12 R.M.S. / Max. value - Time curves

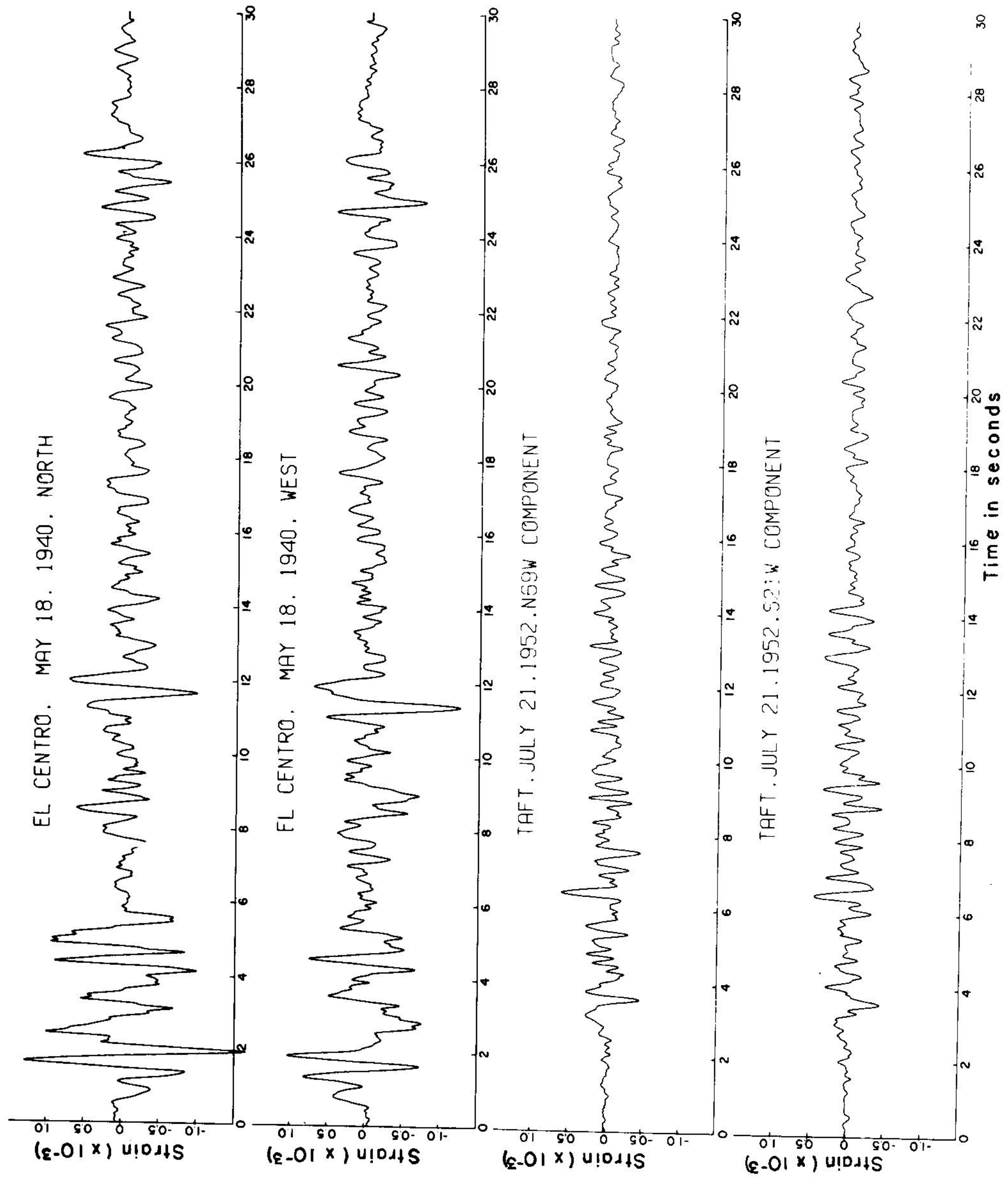

Fig. 13 STRAIN - TIME CURVES AT INTERFACE

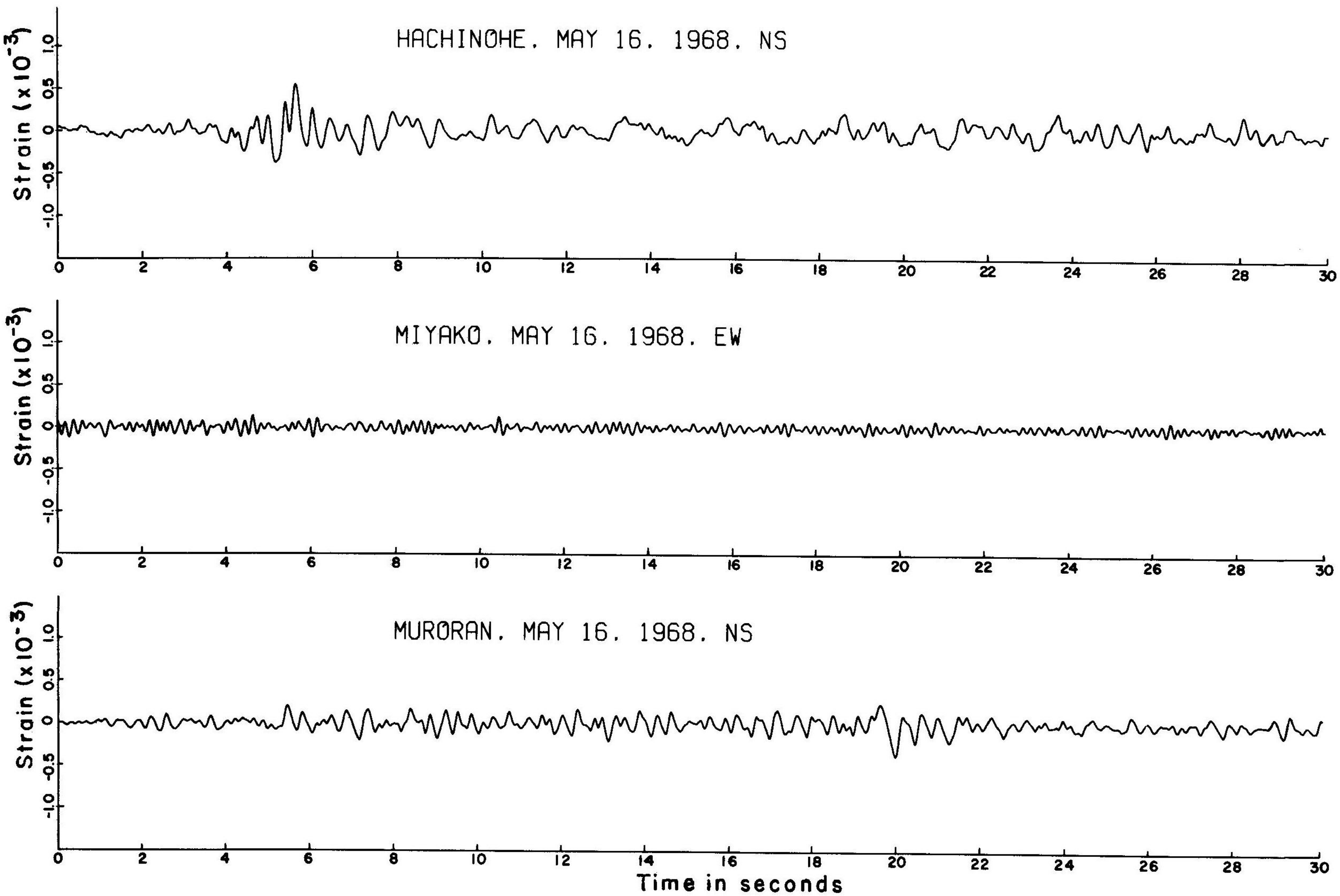

Fig. 14 Strain-Time Curves (at interface)